



Group Decision Making Methods Based on Multi Q-Fuzzy Soft Interval Set

Fatma Adam and Nasruddin Hassan *

*School of Mathematical Sciences, Faculty of Science and
Technology, Universiti Kebangsaan Malaysia, Malaysia.*

E-mail: nas@ukm.edu.my

** Corresponding author*

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ABSTRACT

In this article we use the concept of a multi Q-fuzzy set for defining a multi Q-fuzzy soft interval set and propose properties and corresponding theorems. An adjustable approach is presented by means of average probability. Finally, a novel concept called soft interval cut-sets is proposed for dealing with decision making problems based on multi Q-fuzzy soft interval set, followed by a few illustrative examples.

Keywords: Average probability, interval set, interval cut-set, multi q-fuzzy set, soft set, soft interval set.

1. Introduction

The past decade has seen the rapid development of set theory in many different fields such as fuzzy sets by Varnamkhasti and Hassan (2012) in neuro-genetics and neuro-fuzzy inference (2013). Molodtsov (1999) presented soft set as a completely generic mathematical instrument for modelling uncertainties. Many efforts have been devoted to further generalizations and extensions of Molodtsov's soft sets, notably on the soft solution of soft set theory in decision-making problems by Hakim et al. (2014). Zhu and Wen (2013) proposed some operations in soft sets, followed by Feng et al. (2010) on adjustable decision making scheme using fuzzy soft sets, Tahat et al. (2015) on ordering of vague soft set, and Alzu'bi et al. (2015) on vague soft set relation mapping.

Alkhazaleh et al. (2011) introduced fuzzy parameterized interval-valued fuzzy soft set theory followed by multiparameterized soft set by Salleh et al. (2012), soft intuitionistic fuzzy sets by Alhazaymeh et al. (2012), while Alhazaymeh and Hassan (2012a, 2012b, 2012c) applied vague soft sets to decision making, and further explored interval-valued vague soft set (2013a, 2013c), generalized vague soft expert set (2013b, 2014a, 2014b, 2014c), vague soft multiset theory (2014d), and vague soft set relations and functions (2015). Al-Quran and Hassan (2016a, 2016b) then extended the work of Hassan and Alhazaymeh (2013) on vague soft expert set to neutrosophic versions. Thus decision making are not necessarily in certain environments (Hassan and Ayop (2012), Hassan and Halim (2012), Hassan and Loon (2012), Hassan and Sahrin (2012), Hassan et al. (2012)). The notion of interval sets was proposed by Yao (1993) as a mathematical tool for dealing with qualitative information. The combination of interval set and soft set models was recently discussed by Qin et al. (2013) followed by Zhang (2014b) on interval soft sets and their applications. This work aims to extend the study by Adam and Hassan (2014b, 2014e) on Q-fuzzy soft sets, followed by Q-fuzzy soft matrix ((2014c, 2014d) and multi Q-fuzzy soft sets (2014a, 2015, 2016a, 2016b), to multi Q-fuzzy soft interval sets and study their properties. In this work, we establish the concept of multi Q-fuzzy interval soft set to improve the reasonability of decision making in reality, and handling complex problems which have lower bounds and upper bounds of parameter approximations with multi dimensional characterization properties. We propose an adjustable approach by using average probability of multi Q-fuzzy soft interval sets. Different kinds of soft interval cut-sets will be considered and illustrated by some concrete examples.

2. Preliminaries

In this section, we briefly review some basic concepts concerning soft expert set and soft interval set.

Definition 2.1. *Adam and Hassan (2014d) Let I be a unit interval $[0, 1]$, k be a positive integer, U be a universal set and Q be a non-empty set. A multi Q-fuzzy set \tilde{A}_Q in U and Q is a set of ordered sequences $\tilde{A}_Q = \{((u, q), (\mu_1(u, q),$*

$\mu_2(u, q), \dots, \mu_k(u, q)) : u \in U, q \in Q\}$, where

$$\mu_i(u, q) \in I, \text{ for all } i = 1, 2, \dots, k$$

The function $(\mu_1(u, q), \mu_2(u, q), \dots, \mu_k(u, q))$ is called the membership function of multi Q-fuzzy set A_Q ; and $\mu_1(u, q) + \mu_2(u, q) + \dots + \mu_k(u, q) \leq 1$, k is called the dimension of A_Q . In other words, if the sequences of the membership functions have only k -terms (finite number of terms) the multi Q-fuzzy set is a function from $U \times Q$ to I^k such that for all $(u, q) \in U \times Q$, $\mu_{A_Q} = (\mu_1(u, q), \mu_2(u, q), \dots, \mu_k(u, q))$. The set of all multi Q-fuzzy sets of dimension k in U and Q is denoted by $M^k QF(U)$.

Definition 2.2. Qin et al. (2013) A pair (F, E) is called a soft interval set (over U), where $A \subseteq E$ and F is a mapping given by

$$F : A \longrightarrow I(2^U).$$

Definition 2.3. Maji et al. (2001) Let U be an initial universal set and let E be a set of parameters. Let I^U denote the power set of all fuzzy subsets of U . Let $A \subseteq E$. A pair (F, E) is called a fuzzy soft set over U where F is a mapping given by

$$F : A \rightarrow I^U.$$

3. Multi Q-fuzzy Soft Interval Set

In this section, we introduce the definition of a multi Q-fuzzy soft interval set, and its basic operations such as null, whole, relative complement, union and intersection.

Definition 3.1. Let U be a universal set, E be a set of parameters, and Q be a non-empty set. Let $I(MQF)$ denote the interval sets of all multi Q-fuzzy subsets of U and $A \subseteq E$. A pair (F_q, A) is called a multi Q-fuzzy soft interval set (in short $(MQF)SI$ set) over U where F_q is a mapping given by

$$F_q : A \rightarrow I(MQF) \text{ such that } F_q(x) = [\emptyset, \emptyset] \text{ if } x \notin A.$$

Here a multi Q-fuzzy soft interval set can be represented by the set of ordered pairs

$$(F_q, A) = \{(x, [f_q(x)_-, f_q(x)_+]) : x \in A, f_q \in (MQF)\}.$$

In other words, multi Q-fuzzy soft interval set over U is a parameterized family of interval sets of multi Q-fuzzy subset of U . Note that the set of all multi Q-fuzzy soft interval set over U will be denoted by $(MQF)SI$.

We will illustrate the advantages of our proposed method using multi Q-fuzzy soft interval set as compared to that of fuzzy soft set as proposed by Maji et al. (2001) in the following two examples.

Example 3.1. Assume a company wants to fill a position. There are five candidates to choose from the set of alternatives, $U = \{u_1, u_2, u_3, u_4, u_5\}$. The hiring committee considers a set of parameters, $E = \{x_1, x_2, x_3\}$. For $i = 1, 2, 3$ the parameters x_i stand for young age, experience and fluent, respectively. Suppose they decide to a chosen subset $A = \{x_2, x_3\}$ of E . The fuzzy soft set can describe this problem as follows.

$$F(x_2) = \{(u_1, 0.4), (u_2, 0.6), (u_3, 0.7), (u_4, 0.5), (u_5, 0.1)\},$$

$$F(x_3) = \{(u_1, 0.6), (u_2, 0.9), (u_3, 0.1), (u_4, 0.2), (u_5, 0.3)\}.$$

In Example 3.1 above, it will be difficult to explain the universal U in more detail with only one membership function, especially when there are many parameters involved. Now we illustrate the notion of $(MQF)SI$ by an example to show its ability to handle a problem with many parameters as follows.

Example 3.2. Assume a company wants to fill a position. There are five candidates to choose from the set of alternatives, $U = \{u_1, u_2, u_3, u_4, u_5\}$, and the qualification $Q = \{q, p\}$. The hiring committee consider a set of parameters, $E = \{x_1, x_2, x_3, x_4, x_5\}$. For $i = 1, 2, 3, 4, 5$, the parameters x_i stand for young age, experience, fluent, good-looking and friendly, respectively. Suppose they decide to a chosen subset $A = \{x_2, x_3, x_5\}$ of E .

$F_q(x_2) = \{((u_1, q), 0.3, 0.4, 0.2), ((u_1, p), 0.3, 0, 0.2), ((u_3, q), 0.3, 0.6, 0.1), ((u_3, p), 0.6, 0.3, 0.1)\}, \{((u_1, q), 0.3, 0.4, 0.5), ((u_1, p), 0.4, 0, 0.2), ((u_2, q), 0.1, 0.2, 0.2), ((u_2, p), 0.3, 0.1, 0.1), ((u_3, q), 0.3, 0.6, 0.1), ((u_3, p), 0.6, 0.3, 0.1)\}$,
 $F_q(x_3) = \{((u_2, q), 0.2, 0.7, 0.1), ((u_2, p), 0.1, 0.1, 0.2), ((u_3, q), 0.2, 0.6, 0.1), ((u_3, p), 0.4, 0.4, 0.1)\}, \{((u_1, q), 0.3, 0.4, 0.2), ((u_1, p), 0.3, 0.4, 0.2), ((u_2, q), 0.2, 0.7, 0.1), ((u_2, p), 0.1, 0.1, 0.2), ((u_3, q), 0, 2, 0.6, 0.1), ((u_3, p), 0.4, 0.4, 0.1)\}$.
 $F_q(x_5) = \{((u_2, q), 0.2, 0.7, 0.1), ((u_2, p), 0.1, 0.1, 0.2)\}, \{((u_2, q), 0, 2, 0.6, 0.1), ((u_2, p), 0.4, 0.4, 0.1), ((u_5, q), 0.3, 0.4, 0.2), ((u_5, p), 0.3, 0.4, 0.2)\}$. Then
 $(F_q, A) = \{x_2, [\{((u_1, q), 0.3, 0.4, 0.2), ((u_1, p), 0.3, 0.4, 0.2), ((u_3, q), 0.3, 0.6, 0.1), ((u_3, p), 0.6, 0.3, 0.1)\}, \{((u_1, q), 0.3, 0.4, 0.5), ((u_1, p), 0.4, 0, 0.2), ((u_2, q), 0.1, 0.2, 0.2), ((u_2, p), 0.3, 0.1, 0.1), ((u_3, q), 0.3, 0.6, 0.1), ((u_3, p), 0.6, 0.3, 0.1)\}]\},$
 $\{x_3, [\{((u_2, q), 0.2, 0.7, 0.1), ((u_2, p), 0.1, 0.1, 0.2), ((u_3, q), 0.2, 0.6, 0.1), ((u_3, p), 0.4, 0.4, 0.1)\}, \{((u_1, q), 0.3, 0.4, 0.2), ((u_1, p), 0.3, 0.4, 0.2), ((u_2, q), 0.2, 0.7, 0.1), ((u_2, p), 0.1, 0.1, 0.2), ((u_3, q), 0, 2, 0.6, 0.1), ((u_3, p), 0.4, 0.4, 0.1)\}]\}$ $\} \cup \{x_5, [\{((u_2, q), 0.2, 0.7, 0.1), ((u_2, p), 0.1, 0.1, 0.2)\}, \{((u_2, q), 0, 2, 0.6, 0.1), ((u_2, p), 0.4, 0.4, 0.1), ((u_5, q), 0.3, 0.4, 0.2), ((u_5, p), 0.3, 0.4, 0.2)\}]\}$ is a multi Q -fuzzy soft interval set.

This multi Q -fuzzy soft interval set gives us a collection of approximate descriptions of an object. $F_q(x_2)$ means candidates u_1 and u_3 with qualification Q are experienced, while candidates u_4 and u_5 are not. $F_q(x_3)$ means candidates u_2 and u_3 are fluent, while u_1 is possibly fluent. $F_q(x_5)$ means candidates u_2 is friendly, while candidates u_1, u_3 and u_4 are not.

In order to give a deeper insight, we propose the following definitions.

- Definition 3.2.** 1. (F_q, A) is called a null multi Q-fuzzy soft interval set (with respect to the parameter set A), denoted by $(F_q, A)_\emptyset$ if $F_q(\mu)(e) = [\mu f_q(x)_- = \emptyset, \mu f_q(x)_+ = \emptyset]$ for all $x \in A$.
2. (F_q, A) is called a relative whole multi Q-fuzzy soft interval set (with respect to the parameter A), denoted by $(F_q, A)_U$ if $F_q(\mu)(e) = [\mu f_q(x)_- = U, \mu f_q(x)_+ = U]$ for all $x \in A$.

Definition 3.3. Let $(F_q, A), (H_q, B) \in (MQF)SI$, then $(F_q, A) \subseteq (H_q, B)$, if and only if $A \subseteq B$ and $F_q(x) \sqsubseteq H_q(x)$ such that $\mu f_q(x)_- \leq \mu g_q(x)_-$, and $\mu f_q(x)_+ \leq \mu g_q(x)_+$ for all $x \in A$.

Definition 3.4. Let (F_q, A) and (G_q, B) be two multi Q-fuzzy soft interval sets over a common universe U .

1. The extended union of (F_q, A) and (G_q, B) denoted by $(F_q, A) \cup_e (G_q, B)$ is the multi Q-fuzzy soft interval set (H_q, C) , where $C = A \cup B$, and

$$H_q(e) = \begin{cases} F_q(e), & \text{if } e \in A - B ; \\ G_q(e), & \text{if } e \in B - A ; \\ F_q(e) \sqcup G_q(e), & \text{if } e \in A \cap B. \end{cases}$$

2. The extended intersection of (F_q, A) and (G_q, B) denoted by $(F_q, A) \cap_e (G_q, B)$ is the multi Q-fuzzy soft interval set (H_q, C) , where $C = A \cap B$, and

$$H_q(e) = \begin{cases} F_q(e), & \text{if } e \in A - B ; \\ G_q(e), & \text{if } e \in B - A ; \\ F_q(e) \sqcap G_q(e), & \text{if } e \in A \cap B. \end{cases}$$

3. The restricted union of (F_q, A) and (G_q, B) denoted by $(F_q, A) \cup_r (G_q, B)$ is the multi Q-fuzzy soft interval set (H_q, C) , where $C = A \cap B$, and

$$H_q(e) = F_q(e) \sqcup G_q(e) = [\max(\mu f_{q-}, \mu g_{q-}), \max(\mu f_{q+}, \mu g_{q+})] \text{ for every } e \in C.$$

4. The restricted intersection of (F_q, A) and (G_q, B) denoted by $(F_q, A) \cap_r (G_q, B)$ is the multi Q-fuzzy soft interval set (H_q, C) , where $C = A \cap B$, and $H_q(e) = F_q(e) \sqcap G_q(e) = [\min(\mu f_{q-}, \mu g_{q-}), \min(\mu f_{q+}, \mu g_{q+})]$ for every $e \in C$.

Definition 3.5. The relative complement of a multi Q-fuzzy soft interval set (F_q, A) is denoted by $(F_q, A)^r$ and is defined by $(F_q, A)^r = (F_q^r, A)$ where $F_q^r : A \rightarrow I(MQF)$ is a mapping given by $F_q^r(e) = \neg F_q(\mu)(e) = [1 - \mu f_q(x)_-, 1 - \mu f_q(x)_+] \forall e \in A$.

Using the above definitions, we establish the following theorems.

Theorem 3.1. Let $(F_q, A), (G_q, B) \in (MQF)SI$. Then $(F_q, A) \subseteq (G_q, B)$, if and only if $(F_q, A) \cup_e (G_q, B) = (G_q, B)$.

Proof. Suppose that $(F_q, A) \cup_e (G_q, B) = (H_q, A \cup B)$.

If $F_q(\mu)(a) \sqsubseteq G_q(\mu)(a)$ implies that $\mu_{f_{q-}} \leq \mu_{g_{q-}}, \mu_{f_{q+}} \leq \mu_{g_{q+}}$ then $A \subseteq B$ and hence $A \cup B = B$. For each $a \in B$, if $a \notin A$ then $H_q(\mu)(a) = G_q(\mu)(a)$, while if $a \in A$ then $F_q(\mu)(a) \sqcup G_q(\mu)(a) = [\max(\mu_{f_{q-}}, \mu_{g_{q-}}), \max(\mu_{f_{q+}}, \mu_{g_{q+}})] = G_q(\mu)(a)$ by $F_q(\mu)(a) \sqsubseteq G_q(\mu)(a)$. Thus $(F_q, A) \cup_e (G_q, B) = (H_q, A \cup B)$. Conversely, let $(F_q, A) \cup_e (G_q, B) = (G_q, B)$, $A \subseteq B$. For each $a \in A$, by $F_q(\mu)(a) \sqcup G_q(\mu)(a) = [\max(\mu_{f_{q-}}, \mu_{g_{q-}}), \max(\mu_{f_{q+}}, \mu_{g_{q+}})] = G_q(\mu)(a)$, implies that $F_q(\mu)(a) \sqsubseteq G_q(\mu)(a)$. Hence $(F_q, A) \subseteq (G_q, B)$. \square

Theorem 3.2. Let $(F_q, A), (G_q, B) \in (MQF)SI$. Then $(F_q, A) \subseteq (G_q, B)$, if and only if $(F_q, A) \cap_e (G_q, B) = (F_q, A)$.

Proof. The theorem could be proven in the same way as the proof in Theorem 3.1. \square

Theorem 3.3. Let $(F_q, A), (G_q, B) \in (MQF)SI$. Then the union and intersection of them are $(MQF)SI$ if $(F_q, A) \subseteq (G_q, B)$ or $(G_q, A) \subseteq (F_q, B)$.

Proof. Let $(F_q, A) \subseteq (G_q, B)$. Then by Theorem 3.1 we have $(F_q, A) \cup_e (G_q, B) = (G_q, B)$. Thus $(F_q, A) \cup_e (G_q, B)$ is $(MQF)SI$. In a similar fashion, we can deduce $(F_q, A) \cap_e (G_q, B)$ is $(MQF)SI$. \square

Theorem 3.4. Let $(F_q, A), (G_q, B) \in (MQF)SI$ be two multi Q -fuzzy soft interval sets over the same universe U and $A \cap B \neq \emptyset$. Then

1. $((F_q, A) \cup_r (G_q, B))^r = (F_q, A)^r \cap_r (G_q, B)^r$.
2. $((F_q, A) \cap_r (G_q, B))^r = ((F_q, A)^r \cup_r (G_q, B))^r$.
3. $((F_q, A) \cup_e (G_q, B))^r = (F_q, A)^r \cap_e (G_q, B)^r$.
4. $((F_q, A) \cap_e (G_q, B))^r = ((F_q, A)^r \cup_e (G_q, B))^r$.

Proof. 1. Let $(F_q, A) \cup_r (G_q, B) = (H_q, A \cap B)$. Thus $(F_q, A)^r \cap_r (G_q, B)^r = (K_q, A \cap B)^r$. It follows that $((F_q, A) \cup_r (G_q, B))^r = (H_q^r, A \cap B)$. For all $a \in A \cap B$, we have $H_q^r(\mu)(a) = \neg H_q^r(\mu)(a) = \neg(F_q(\mu)(a) \sqcup G_q(\mu)(a)) = \neg[\max(\mu_{f_{q-}}, \mu_{g_{q-}}), \max(\mu_{f_{q+}}, \mu_{g_{q+}})] = [\min(1 - \mu_{f_{q-}}, 1 - \mu_{g_{q-}}), \min(1 - \mu_{f_{q+}}, 1 - \mu_{g_{q+}})] = F_q^r(a) \cap G_q^r(a) = K_q(a)$, and consequently $((F_q, A) \cup_r (G_q, B))^r = (F_q, A)^r \cap_r (G_q, B)^r$.

2. From (1) we have $((F_q, A)^r \cup_r (G_q, B)^r)^r = ((F_q, A)^r)^r \cap_r ((G_q, B)^r)^r = (F_q, A) \cap_r (G_q, B)$, and hence $((F_q, A) \cap_r (G_q, B))^r = (((F_q, A)^r \cup_r (G_q, B)^r)^r)^r = (F_q, A)^r \cup_r (G_q, B)^r$,

3. Let $(F_q, A) \cup_e (G_q, B) = (H_q, A \cup B)$. Thus $(F_q, A)^r \cap_e (G_q, B)^r = (K_q, A \cup B)$. It follows that $((F_q, A) \cup_e (G_q, B))^r = (H_q^r, A \cup B)$. For all $a \in A \cup B$;
- (a) If $a \in A - B$ then $H_q^r(\mu)(a) = \neg H_q(\mu)(a) = \neg F_q(\mu)(a) = F_q^r(a) = K_q(a)$,
 - (b) If $a \in B - A$ then $H_q^r(\mu)(a) = \neg H_q(\mu)(a) = \neg G_q(\mu)(a) = G_q^r(a) = K_q(a)$,
 - (c) If $a \in A \cap B$ then $H_q^r(\mu)(a) = \neg H_q(\mu)(a) = \neg(F_q(a) \sqcup G_q(a)) = [\min(\mu_{f_{q-}}, \mu_{g_{q-}}), \min(1 - \mu_{f_{q+}}, 1 - \mu_{g_{q+}})] = F_q^r(a) \sqcap G_q^r(a) = K_q(a)$,
Thus we have $((F_q, A) \cup_e (G_q, B))^r = (F_q, A)^r \cap_e (G_q, B)^r$.
4. From (3) we have $((F_q, A)^r \cup_e (G_q, B)^r)^r = ((F_q, A)^r \cap_e ((G_q, B)^r)^r = (F_q, A) \cap_e (G_q, B)$. It follows that $((F_q, A) \cap_e (G_q, B))^r = (((F_q, A)^r \cup_e (G_q, B)^r)^r)^r = (F_q, A)^r \cup_e (G_q, B)^r$.

□

Definition 3.6. Let (F_q, A) and (G_q, B) be two multi Q-fuzzy soft interval sets over the universe U and non empty set Q . (F_q, A) is said to be equal to (G_q, B) , denoted by $(F_q, A) \approx (G_q, B)$ if for all $a \in A \cup B$, while $a \in A \cap B$ implies $F_q(\mu)(a) = G_q(\mu)(a)$, $a \in A - B$ implies $F_q(\mu)(a) = [\mu_{f_{q-}} = \emptyset, \mu_{f_{q+}} = \emptyset]$ and $a \in B - A$ implies $G_q(\mu)(a) = [\mu_{g_{q-}} = \emptyset, \mu_{g_{q+}} = \emptyset]$.

In view of the above Definition 3.3 we prove the following result.

Theorem 3.5. Let (F_q, A) and (G_q, B) be two multi Q-fuzzy soft interval sets over the universe U and non empty set Q . Then $(F_q, A) \approx (G_q, B)$ if and only if $(F_q, A) \cup_e (G_q, B) \approx (F_q, A) \cap_r (G_q, B)$.

Proof. Suppose that $(F_q, A) \cup_e (G_q, B) = (H_q, A \cup B)$ and $(F_q, A) \cap_r (G_q, B) = (T_q, A \cap B)$. Let $(F_q, A) \approx (G_q, B)$. For the case $a \in A \cap B$ we can immediately deduce that $F_q(\mu)(a) = G_q(\mu)(a)$, by Definition 3.6. Hence $H_q(\mu)(a) = [\max(\mu_{f_{q-}}, \mu_{g_{q-}}), \max(\mu_{f_{q+}}, \mu_{g_{q+}})] = [\min(\mu_{f_{q-}}, \mu_{g_{q-}}), \min(\mu_{f_{q+}}, \mu_{g_{q+}})] = T_q(\mu)(a)$.

For the other case $a \in A \cup B - A \cap B$, consider $a \in A - B$. Then $F_q(\mu)(a) = [\mu_{f_{q-}} = \emptyset, \mu_{f_{q+}} = \emptyset]$, implies $H_q(\mu)(a) = F_q(\mu)(a) = [\emptyset, \emptyset]$, whereas $G_q(\mu)(a) = [\mu_{g_{q-}} = \emptyset, \mu_{g_{q+}} = \emptyset]$, implies $H_q(\mu)(a) = G_q(\mu)(a) = [\emptyset, \emptyset]$.

Thus $(F_q, A) \cup_e (G_q, B) = (F_q, A) \cap_r (G_q, B)$.

Conversely, let us consider $(F_q, A) \cup_e (G_q, B) = (F_q, A) \cap_r (G_q, B)$. For all $a \in A \cap B$, we have $H_q(\mu)(a) = [\max(\mu_{f_{q-}}, \mu_{g_{q-}}), \max(\mu_{f_{q+}}, \mu_{g_{q+}})] = [\min(\mu_{f_{q-}}, \mu_{g_{q-}}), \min(\mu_{f_{q+}}, \mu_{g_{q+}})]$ and hence $H_q(a) = F_q(a)$. If $a \in A - B$, it follows that $a \in A \cup B$ and $a \notin A \cap B$, and hence $F_q(\mu)(a) = H_q(\mu)(a) = [\mu_{f_{q-}} = \emptyset, \mu_{f_{q+}} = \emptyset]$. If $a \in B - A$, $G_q(\mu)(a) = [\mu_{g_{q-}} = \emptyset, \mu_{g_{q+}} = \emptyset]$ can be similarly proven. Therefore, we can deduce that $(F_q, A) \approx (G_q, B)$. □

As an immediate consequence of the above theorem, we get the following result.

Theorem 3.6. *Let (F_q, A) and (G_q, B) be two multi Q -fuzzy soft interval set over the universe U and non empty set Q . Then $(F_q, A) \approx (G_q, B)$ if and only if $(F_q, A) \cup_r (G_q, B) \approx (F_q, A) \cap_e (G_q, B)$.*

Proof. The proof is similar to the proof of Theorem 3.5. □

4. Applications to Decision Making Problems

The theory of soft set has been applied to data analysis and decision systems based on large data sets. Kong et al. (2014) presented soft set under incomplete information. They put forward improved data analysis approaches for soft set under incomplete information using simplified probability. Zhang (2014a) studied a weighted attribute decision making approach in incomplete soft set. Motivated by these points, we will explore in this section an application of multi Q -fuzzy soft interval set in a decision making problem by generalizing Kong et al. (2014) method to be compatible with our work. The problem we consider is as follows.

Example 4.1. *Suppose that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is the set of the attractiveness of cell phones which a person is going to buy under consideration, $Q = \{p, q\}$ is a set of brand names, $A = \{e_1, e_2, e_3, e_4\}$ is the set of parameters, where e_1 stands for the parameter color which consists of black, white and blue, e_2 stands for the parameter material which is made from plastic, liquid crystal and metal, e_3 stands for the parameter price which can be expensive, reasonable and cheap, and e_4 stands for weight which can be heavy, light and very light. Suppose that $A = \{e_2, e_3, e_4\}$ is the set of parameters concerned, thus the multi Q -fuzzy interval soft set (F_q, A) can describe the attractiveness of the cell phones as follows.*

$$\begin{aligned} (F_q, A) = \{ & (x_2, [\{(u_1, q), 0.3, 0.4, 0.2\}, \{(u_1, p), 0.3, 0.4, 0.2\}, \\ & \{(u_3, q), 0.3, 0.6, 0.1\}, \{(u_3, p), 0.6, 0.3, 0.1\}\}, \{(u_2, q), 0.1, 0.2, 0.2\}, \\ & \{(u_2, p), 0.3, 0.1, 0.1\}, \{(u_3, q), 0.3, 0.6, 0.1\}, \{(u_3, p), 0.6, 0.3, 0.1\}\}], \\ & (x_3, [\{(u_2, q), 0.2, 0.7, 0.1\}, \{(u_2, p), 0.1, 0.1, 0.2\}, \{(u_3, q), 0.2, 0.6, 0.1\}, \\ & \{(u_3, p), 0.4, 0.4, 0.1\}\}, \{(u_1, q), 0.3, 0.4, 0.2\}, \{(u_1, p), 0.3, 0, 0.2\}, \\ & \{(u_2, q), 0.2, 0.7, 0.1\}, \{(u_2, p), 0.1, 0.1, 0.2\}, \{(u_3, q), 0.2, 0.6, 0.1\}, \\ & \{(u_3, p), 0.4, 0.4, 0.1\}\}], (x_4, [\{(u_2, q), 0.2, 0.7, 0.1\}, \{(u_2, p), 0.1, 0.1, 0.2\}, \\ & \{(u_4, q), 0, 2, 0.6, 0.1\}, \{(u_4, p), 0.4, 0.4, 0.1\}\}, \{(u_5, q), 0.3, 0.4, 0.2\}, \\ & \{(u_5, p), 0.3, 0.4, 2\}\}]\}. \end{aligned}$$

Following the idea of incomplete information system, we represent multi Q -fuzzy soft interval set (F_q, A) in a binary table and its entries $(u_i, q_j) \in F_q(x)$ implies $(u_i, q_j) = \mu_{f_q(u_i, q_j)}$, $(u_i, q_j) \in F_q(x)_+ - F_q(x)_-$ implies $(u_i, q_j) = *$, and

$(u_i, q_j) \in 1 - F_q(x)_+$ implies $(u_i, q_j) = 0$. $(u_i, q_j) = *$ means we are not sure whether $(u_i, q_j) \in F_q(x)$ or not.

The previous example can be represented in the following Table 1.

Since the tables contain incomplete information, assume that P_x be the probability of an object in $F_q(x)_+ - F_q(x)_-$ belonging to $F_q(x)$, with respect to the parameter x . For multi Q-fuzzy soft interval set we can define

$$p_x = \frac{|F_q(x)_-|}{|F_q(x)_-| + |1 - F_q(x)_+|}, \quad x \in A.$$

We are now able to compute the average probability of each parameter $x \in A$ as

$$P_{x_2} = 0.27, \quad P_{x_3} = 0.20 \quad \text{and} \quad P_{x_4} = 0.43.$$

The previous Table 1 can be represented in the following Table 2 using average probability of each parameter.

Table 1: The tabular representation of the multi Q-fuzzy soft interval set (F_q, A)

(u_i, q_i)	x_2	x_3	x_4
(u_1, q)	(0.3, 0.4, 0.2)	*	0
(u_1, p)	(0.3, 0, 0.2)	*	0
(u_2, q)	*	(0.2, 0.7, 0.1)	(0.2, 0.7, 0.1)
(u_2, p)	*	(0.1, 0.1, 0.2)	(0.1, 0.1, 0.2)
(u_3, q)	(0.3, 0.6, 0.1)	(0.2, 0.6, 0.1)	0
(u_3, p)	(0.6, 0.3, 0.1)	(0.4, 0.4, 0.1)	0
(u_4, q)	0	0	(0.2, 0.6, 0.1)
(u_4, p)	0	0	(0.4, 0.4, 0.1)
(u_5, q)	0	0	*
(u_5, p)	0	0	*

Table 2: (F_q, A) using average probability of each parameter

(u_i, q_j)	x_2	x_3	x_4
(u_1, q)	(0.3, 0.4, 0.2)	0.20	0
(u_1, p)	(0.3, 0, 0.2)	0.20	0
(u_2, q)	0.27	(0.2, 0.7, 0.1)	(0.2, 0.7, 0.1)
(u_2, p)	0.27	(0.1, 0.1, 0.2)	(0.1, 0.1, 0.2)
(u_3, q)	(0.3, 0.6, 0.1)	(0.2, 0.6, 0.1)	0
(u_3, p)	(0.6, 0.3, 0.1)	(0.4, 0.4, 0.1)	0
(u_4, q)	0	0	(0.2, 0.6, 0.1)
(u_4, p)	0	0	(0.4, 0.4, 0.1)
(u_5, q)	0	0	0.43
(u_5, p)	0	0	0.43

Suppose w_k is the relative weight of parameter $x \in A$ in the interval $[1, 1]$. The choice value c_k of object $(u_i, q_j)_k$ will be computed by $c_k = \sum_k^{i \times j} w_k \mu_i(u_i, q_j)_k$ where $\mu_i(u_i, q_j)_k$ are entries in the extended tabular representation of (F_q, A) . Assume that the weights of each of the parameters are

as follows:

$$x_2 = \{-0.1, 0.3, 0.5\}, x_3 = \{0.1, 0.2, 0.3\} \text{ and } x_4 = \{-0.2, 0.1, 0.6\}.$$

The extended representation of (F_q, A) with weights are given in Table 3.

Table 3: The tabular representation of (F_q, A) with weights and choice value of each object.

(u_i, q_j)	$x_1\{-0.1, 0.3, 0.5\}$,	$x_3\{0.1, 0.2, 0.3\}$	$x_4\{-0.2, 0.1, 0.6\}$	c_i
(u_1, q)	0.19	0.12	0	0.31
(u_1, p)	0.07	0.12	0	0.19
(u_2, q)	0.189	0.19	0.09	0.469
(u_2, p)	0.189	0.09	0.09	0.369
(u_3, q)	0.2	0.17	0	0.37
(u_3, p)	0.2	0.15	0	0.35
(u_4, q)	0	0	0.08	0.08
(u_4, p)	0	0	0.02	0.02
(u_5, q)	0	0	0.215	0.215
(u_5, p)	0	0	0.215	0.215

The maximum value as shown in Table 3 is $c_3 = 0.469$. Thus (u_2, q) will be chosen.

The following algorithm summarizes the previous decision.

Step 1 Represent the (F_q, A) in tabular form.

Step 2 Compute the average probability P_x for each parameter $x \in A$, such that

$$P_x = \frac{|F_q(x)_-|}{|F_q(x)_-| + |1 - F_q(x)_+|}, x \in A.$$

Step 3 Input the weights w_k of each parameter in A .

Step 4 Find choice values c_k for each object $(h_i, q_j)_k$ where $c_k = \sum_k^{i \times j} w_k \mu_i(u_i, q_j)_k$, and $\mu_i(u_i, q_j)_k$ are entries in the extended tabular representation of (F_q, A) .

Step 5 Find the optimal choice object c_k , for which $c_r = \max_k c_k$. If r has more than one value, then any one of them may be chosen.

5. An Adjustable Approach to $(MQFS)IS$ Based Decision Making

We present an adjustable approach to multi Q-fuzzy soft interval set. This proposal is based on the following new concept called interval cut-set of soft interval sets.

Definition 5.1. Let $\mathfrak{G} = (F_q, A)$ be a multi Q-fuzzy soft interval set, where $A \subseteq E$ and E is the parameter set. For $\lambda = [\lambda_1, \lambda_2] \in [0, 1]$, the different types of interval cut-sets on (MQFS)IS are defined as follows:

$$\begin{aligned} \mathfrak{G}_\lambda^{(1,1)}(a) &= \mathfrak{H}_{[\lambda_1, \lambda_2]} = \{a, F_{q-} \geq \lambda_1, F_{q+} \geq \lambda_2\}, \\ \mathfrak{G}_\lambda^{(1,2)}(a) &= \mathfrak{H}_{[\lambda_1, \lambda_2]} = \{a, F_{q-} \geq \lambda_1, F_{q+} > \lambda_2\}, \\ \mathfrak{G}_\lambda^{(2,1)}(a) &= \mathfrak{H}_{[\lambda_1, \lambda_2]} = \{a, F_{q-} > \lambda_1, F_{q+} \geq \lambda_2\}, \\ \mathfrak{G}_\lambda^{(2,2)}(a) &= \mathfrak{H}_{[\lambda_1, \lambda_2]} = \{a, F_{q-} > \lambda_1, F_{q+} > \lambda_2\}, \\ \mathfrak{G}_\lambda^{(3,3)}(a) &= \mathfrak{H}_{[\lambda_1, \lambda_2]} = \{a, F_{q-} \leq \lambda_1, F_{q+} \leq \lambda_2\}, \\ \mathfrak{G}_\lambda^{(3,4)}(a) &= \mathfrak{H}_{[\lambda_1, \lambda_2]} = \{a, F_{q-} \leq \lambda_1, F_{q+} < \lambda_2\}, \\ \mathfrak{G}_\lambda^{(4,3)}(a) &= \mathfrak{H}_{[\lambda_1, \lambda_2]} = \{a, F_{q-} < \lambda_1, F_{q+} \leq \lambda_2\}, \\ \mathfrak{G}_\lambda^{(4,4)}(a) &= \mathfrak{H}_{[\lambda_1, \lambda_2]} = \{a, F_{q-} < \lambda_1, F_{q+} < \lambda_2\}, \end{aligned}$$

for all $a \in A$, where (i, j) th $[\lambda_1, \lambda_2]$ are the interval cut-sets of the multi Q-fuzzy soft interval set.

A new notion could be obtained by the use of mean value of all lower and upper memberships of multi Q-fuzzy soft interval sets as follows.

Definition 5.2. Let $\mathfrak{G} = (F_q, A)$ be a multi Q-fuzzy soft interval set, where $A \subseteq E$ and E is the parameter set. Based on (F_q, A) we can define the interval mid-set of (MQFS)IS as follows :

$$mid_{\mathfrak{G}}(a) = \left[\frac{1}{|U \times Q|} \sum F_q(a)_-, \frac{1}{|U \times Q|} \sum F_q(a)_+ \right], \tag{1}$$

for all $a \in A$. The multi Q-fuzzy set of $mid_{\mathfrak{G}}$ is called the mid-threshold of the multi Q-fuzzy soft interval set \mathfrak{G} . In addition, the interval cut-set of soft interval set of \mathfrak{G} with respect to the mid-threshold multi Q-fuzzy set, is called the interval mid- soft interval set of \mathfrak{G} and can be simply denoted by (\mathfrak{G}, mid) .

The mean value is easily affected by extreme values. In order to overcome the adverse effect of extreme values, we define the median value to be obtained as follows.

Definition 5.3. Let $\mathfrak{G} = (F_q, A)$ be a multi Q-fuzzy soft interval set, where $A \subseteq E$ and E is the parameter set. Based on (F_q, A) we define the interval median set of (MQFS)IS as follows:

$$med_{\mathfrak{G}}(a) = [med\mu_{F_q}(a)_-, med\mu_{F_q}(a)_+],$$

for all $a \in A$, where $med\mu_{F_q}(a)_-$ and $med\mu_{F_q}(a)_+$ are the median by the ranking of lower and upper multi membership degree of all alternatives according to order from small to large (or from large to small), namely

$$med\mu_{F_q}(a) = \begin{cases} \mu_{f_q(u,q)_{(m+1)/2}}, & \text{if } m \text{ is an odd number ;} \\ (\mu_{f_q(u,q)_{m/2}} + \mu_{f_q(u,q)_{\frac{m}{2}+1}})/2, & \text{if } m \text{ is an even number;} \end{cases} \tag{2}$$

The multi Q-fuzzy set of $med_{\mathfrak{G}}$ is called med-threshold of the multi Q-fuzzy soft interval set \mathfrak{G} . In addition, the interval cut-set of soft interval set of \mathfrak{G} with respect to the med-threshold multi Q-fuzzy set is called the interval med- soft interval set of \mathfrak{G} and can be simply denoted by (\mathfrak{G}, med) .

Let us consider the following example to illustrate the above idea.

Example 5.1. Suppose a study is conducted to select the most attractive city based on a set of parameters. There are five cities to choose from the set of alternatives, $U = \{u_1, u_2, u_3, u_4, u_5\}$ under two conditions $Q = \{q = \text{standard of living}, p = \text{healthcare}\}$. The researchers consider a set of parameters, $E = \{e_1, e_2, e_3, e_4\}$. For $i = 1, 2, 3, 4$ the parameters stand for high, middle, low and very low, respectively. Assume that the researchers decide to a chosen subset $A = \{e_1, e_2\} \subseteq E$. Finally, they apply the following steps:

1. Construct a multi Q-fuzzy soft interval set

$$(F_q, A) = \{ (e_1, [\{((u_1, q), 0.3, 0.4, 0.2), ((u_1, p), 0.3, 0, 0.2), ((u_3, q), 0.3, 0.6, 0.2), ((u_3, p), 0.6, 0.3, 0.2), ((u_4, q), 0, 0.3, 0.5), ((u_4, p), 0, 0.2, 0.3)\}, \{((u_2, q), 0.1, 0.2, 0.1), ((u_2, p), 0.3, 0.1, 0.1), ((u_3, q), 0.3, 0.6, 0.1), ((u_3, p), 0.6, 0.3, 0.1), ((u_4, q), 0.2, 0, 0.1), ((u_4, p), 0, 0.3, 0.4)\}]), (e_2, [\{((u_2, q), 0.2, 0.7, 0.1), ((u_2, p), 0.1, 0.1, 0.2), ((u_3, q), 0.2, 0.6, 0.1), ((u_3, p), 0.4, 0.4, 0.1)\}, \{((u_1, q), 0.3, 0.4, 0.2), ((u_1, p), 0.3, 0, 0.2), ((u_2, q), 0.2, 0.7, 0.1), ((u_2, p), 0.1, 0.1, 0.2), ((u_3, q), 0.2, 0.6, 0.1), ((u_3, p), 0.4, 0.4, 0.1)\}]) \}$$

2. Represent the lower and upper of (F_q, A) in Table 4 and Table 5 using the average mean and median of each parameter by using equations 1 and 2 respectively.

Table 4: The lower and upper of (F_q, A) using average mean and median of parameter e_1 .

(u_i, q_i)	lower	upper
(u_1, q)	(0.3,0.4,0.2)	0
(u_1, p)	(0.3,0,0.2)	0
(u_2, q)	0	(0.1,0.2,0.1)
(u_2, p)	0	(0.3,0.1,0.1)
(u_3, q)	(0.3,0.6,0.2)	(0.3,0.6,0.1)
(u_3, p)	(0.6,0.3,0.2)	(0.6,0.3,0.1)
(u_4, q)	(0,0.3,0.5)	(0.2,0,0.1)
(u_4, p)	(0,0.2,0.3)	(0,0.3,0.4)
(u_5, q)	0	0
(u_5, p)	0	0
$mid_{\mathfrak{G}}(e_1)$	(0.15,0.18,0.16)	(0.15,0.15,0.09)
$med_{\mathfrak{G}}(e_1)$	(0,0.1,0.2)	(0.05,0.05,0.1)

After we have obtained the interval mid-set soft interval set and interval

Table 5: The lower and upper of (F_q, A) using average mean and median of parameter e_2 .

(u_i, q_i)	lower	upper
(u_1, q)	0	(0.3,0.4,0.2)
(u_1, p)	0	(0.3,0,0.2)
(u_2, q)	(0.2,0.7,0.1)	(0.2,0.7,0.1)
(u_2, p)	(0.1,0.1,0.2)	(0.1,0.1,0.2)
(u_3, q)	(0.2,0.6,0.1)	(0.2,0.6,0.1)
(u_3, p)	(0.4,0.4,0.1)	(0.4,0.4,0.1)
(u_4, q)	0	0
(u_4, p)	0	0
(u_5, q)	0	0
(u_5, p)	0	0
$mid\mathfrak{G}(e_2)$	(0.09,0.18,0.05)	(0.15,0.22,0.09)
$med\mathfrak{G}(e_2)$	0	(0.15,0.05,0.1)

Table 6: The choice value of all alternatives using interval mid-set soft interval set.

(u_i, q_j)	e_1	e_2	c_i
(u_1, q)	0	1	0
(u_1, p)	0	0	0
(u_2, q)	0	1	1
(u_2, p)	0	0	0
(u_3, q)	1	1	2
(u_3, p)	1	1	2
(u_4, q)	0	0	0
(u_4, p)	0	0	0
(u_5, q)	0	0	0
(u_5, p)	0	0	0

med-set soft, we can use the choice values to rank the alternatives, namely presenting the soft interval-cut set in tabular form and compute the choice value c_k of $(u_i, q_j)_k$ where $k = 1, 2, \dots, i \times j$, and $j = 1, 2$, and the optimal decision is to select u_r if $c_r = max_k c_k$.

We represent the choice value of all alternatives in Table 6 and Table 7 using interval mid-set and using interval med-set soft interval set. The maximum value as shown in Table 6 is $c_5 = c_6 = 2$, while the maximum value as shown in Table 7 is also the same. Thus the optimal decision is to choose city u_3 as the most attractive to live in based on standard cost of living and health care provisions.

The following algorithm summarizes the previous decision.

Step 1 Represent the (F_q, A) in tabular form.

Step 2 Compute the mid-threshold of the multi Q-fuzzy sets for lower and upper of each parameter as follows:

Table 7: The choice value of all alternatives using interval med-set soft interval set.

(u_i, q_j)	e_1	e_2	c_i
(u_1, q)	0	0	0
(u_1, p)	0	0	0
(u_2, q)	0	1	1
(u_2, p)	0	0	0
(u_3, q)	1	1	2
(u_3, p)	1	1	2
(u_4, q)	0	0	0
(u_4, p)	0	0	0
(u_5, q)	0	0	0
(u_5, p)	0	0	0

$$mid_{\mathfrak{S}}(a) = \left[\frac{1}{|U \times Q|} \sum F_q(a)_-, \frac{1}{|U \times Q|} \sum F_q(a)_+ \right].$$

Step 3 Compute the med-threshold of the multi Q-fuzzy sets for lower and upper of each parameter as follows:

$$med_{\mathfrak{S}}(a) = [med\mu_{F_q}(a)_-, med\mu_{F_q}(a)_+],$$

for all $a \in A$, where

$$med\mu_{F_q}(a) = \begin{cases} \mu_{f_q(u,q)_{(m+1)/2}}, & \text{if } m \text{ is an odd number ;} \\ (\mu_{f_q(u,q)_{m/2}} + \mu_{f_q(u,q)_{(m+1)/2}})/2, & \text{if } m \text{ is an even number;} \end{cases}$$

Step 4 Compute the choice values c_k of $(u_i, q_j)_k$.

Step 5 Find the optimal choice object c_k , for which $c_r = max_k c_k$. If r has more than one value, then any one of them may be chosen.

6. Conclusion

We have defined the concept of multi Q-fuzzy soft interval sets. Based on this concept, some relevant properties and operations of a multi Q-fuzzy soft interval set have been discussed. We have proposed an adjustable approach by using average probability and interval cut-set of multi Q-fuzzy soft interval sets and illustrated these novel methods with some concrete examples. This new proposal proves to be not only suitable but more feasible for some real-life applications of decision making in an imprecise environment.

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